Solution to example problems in the Chapter 5 Notes
Antennas and Propagation

Example 1 - What is the most efficient length of an FM monopole antenna operating in the range 88 – 108 MHz?
What is the most efficient length of an FM dipole antenna operating in the same range?

Let's find the middle frequency first. The middle frequency of the FM band is the average of 88 and 108 MHz:

Middle frequency = (88 + 108)/2 = 196 / 2 = 98 MHz

The wavelength of the middle frequency is $\lambda = \frac{c}{f} = \frac{300\times10^6 \ m/s}{98\times10^6 \ Hz} = \frac{300}{98} = 3.06 \ meters$

The most efficient length the FM monopole antenna is ¼ of the wavelength, which is:

$$\frac{\lambda}{4} = \frac{3.06 \ meters}{4} = 0.765 \ meters$$
$$= (0.765 \ meters)(39.37 \ inches/meter)(1 \ foot/12 \ inches)$$
$$= \frac{(0.765)(39.37)}{12} = \frac{30.13}{12} = 2.51 \ feet = 30 \ inches$$

The most efficient length the FM dipole antenna is ½ of the wavelength, which is:

$$\frac{\lambda}{2} = 2\left(\frac{\lambda}{4}\right) = 2(2.51 \ feet) = 5 \ feet = 60 \ inches$$

Question: What is the beam width of an isotropic antenna? Ans: 180 degrees

Table 1. Beamwidths for selected 12 GHz parabolic reflector antennas.
Extracted Stallings, W., Wireless Communications and Networks, Prentice-Hall, 2002, page 104.

Antenna diameter (m)	**Beamwidth (degrees)**
0.5	3.5
0.75	2.33
1.0	1.75
1.5	1.166
2.0	0.875
2.5	0.7
5.0	0.35

Question: Using Table 1 as a guide, the diameter of a 12 GHz parabolic reflector antenna must be how many wavelengths to obtain a beamwidth of 1 degree?
Ans: Use interpolation:
antenna diameter = 1.5 + (2.0 – 1.5) * (1 - 1.166)/(0.875 - 1.166) = 1.79 meters

Example 2
What is the effective area and gain of a 12 GHz parabolic reflector antenna having a diameter of 2.5 meters?

According to Table 2 in the chapter 5 notes, the effective area of a parabolic reflector is $A_e = 0.56\,A$ where A is the face area. The face area is the area of a circle whose radius is ½ of 2.5 meters:

$$A = \pi r^2 = \pi\left(\frac{2.5}{2}\right)^2 = \pi(1.25)^2 = \pi(1.5625) = 4.909\,m^2$$

The effective area is $A_e = 0.56\,A = (0.56)(4.909) = 2.75\,m^2$

The antenna gain is

$$\begin{aligned} G = \frac{4\pi f^2 A_e}{c^2} &= \frac{4\pi\left(12\times10^9\right)^2(2.75)}{\left(300\times10^6\right)^2} \\ &= 4\pi(2.75)\left(\frac{12\times10^9}{300\times10^6}\right)^2 = 4\pi(2.75)\left(\frac{12000}{300}\right)^2 \\ &= 4\pi(2.75)(40)^2 = 4\pi(2.75)(1600) \\ &= 4\pi(4400) = 55292 \end{aligned}$$

The gain in dB is: $G_{dB} = 10\log_{10} G = 10\log_{10} 55292 = (10)(4.74) = 47.4\ dB$

Related Question: A NASA antenna in the Deep Space Network (DSN) communicates with deep space probes to locations within the solar system, as well as outside the solar system. The diameter of the antenna is 70 meters. What is the gain of this antenna when operating at 1 GHz region?

$$A = \pi r^2 = \pi\left(\frac{70}{2}\right)^2 = \pi(35)^2 = \pi(1225) = 3848\,m^2$$

The effective area is $A_e = 0.56\,A = (0.56)(3848) = 2155\,m^2$

The antenna gain is

$$\begin{aligned} G = \frac{4\pi f^2 A_e}{c^2} &= \frac{4\pi\left(1\times10^9\right)^2(2155)}{\left(3\times10^8\right)^2} \\ &= 4\pi(2155)\left(\frac{1\times10^{18}}{9\times10^{16}}\right) \\ &= 4\pi(2155)\left(\frac{1}{9}\times10^2\right) = 4\pi(2155)(11.11) = 300913 \end{aligned}$$

The gain in dB is: $G_{dB} = 10\log_{10} G = 10\log_{10}(300913) = (10)(5.47) = 54.7\ dB$

Example 3

Suppose you are trying to design a 14 GHz satellite uplink antenna that must have a gain of 60 dB. What is the required diameter of a parabolic reflector that will have this gain?

Start with gain in dB:

$$G_{dB} = 10\log_{10} G$$
$$60 = 10\log_{10} G$$

Divide by 10:

$$6 = \log_{10} G$$

Apply anti log to compute gain: $G = 10^6$

Take the gain formula with effective area: $G = \dfrac{4\pi f^2 A_e}{c^2}$

Rearrange to solve for effective area:

$$A_e = \frac{G c^2}{4\pi f^2}$$
$$= \frac{10^6 \left(300\times10^6\right)^2}{4\pi\left(14\times10^9\right)^2}$$
$$= 36.54\ m^2$$

The effective area of a parabolic reflector can be expressed as: $A_e = 0.56\,A$

Solve for actual face area:

$$A = \frac{A_e}{0.56} = \frac{36.54}{0.56} = 65.25\ m^2$$

The area of a circle is:

$$A = \pi r^2$$
$$65.25 = \pi r^2$$

Divide by π to get:

$$\frac{65.25}{\pi} = r^2$$
$$r^2 = \frac{65.25}{\pi} = 20.77$$

Perform square root:

$$\frac{65.25}{\pi} = r^2$$
$$r = \sqrt{20.77} = 4.56\ meters$$

The diameter is twice the radius

$$diameter = 2r = (2)(4.56\ meters) = 9.11\ meters$$

Example 4: A sailor stands 100 feet above the sea surface and spies land. How far is it to the land in miles? Calculate using (4) and (5).

Using (5), the distance in miles is $d = \sqrt{1.5h} = \sqrt{1.5 \times 100} = \sqrt{150} = 12.25\ miles$

Now we convert feet to meters:
meters = (feet * 0.3048 meters/ft)

meters = (100 feet * 0.3048 meters/ft)
meters = (100 *0.3048) = 30.48 meters

Then use (6) to compute km:

$$d = 3.57\sqrt{h} = 3.57\sqrt{30.48} = 19.71\ km$$

Convert km to miles: miles = km / 1.609344 = 19.71 / 1.609344 =12.25 miles

Example 5: A sailor stands 100 feet above the sea surface and spies land. He has a 100 MHz radio receiver with him. How far can he receive a radio signal sent from the sea surface?

Now we convert feet to meters:
meters = (100 feet * 0.3048 meters/ft) = 30.48 meters

Then use (6) to compute km:

$$d = 3.57\sqrt{\frac{4}{3}h} = 3.57\sqrt{\frac{4}{3}30.48} = 3.57\sqrt{40.64} = 22.76\ km$$

Convert km to miles: miles = km / 1.609344 = 22.76 / 1.609344 = 14.14 miles

Example 6: Two antennas are used to create a microwave link. One antenna is at the top of a 76 meter tall tower. The other antenna will be placed at the top of a 50 meter tower. How far can the towers be separated and still create a viable microwave link?

$$d = 3.57\left(\sqrt{\frac{4}{3}h_1} + \sqrt{\frac{4}{3}h_2}\right) = 3.57\left(\sqrt{\frac{4}{3}76} + \sqrt{\frac{4}{3}50}\right)$$

$$= 3.57(18.23) = 65\ km$$

Convert km to miles: miles = 65 km / 1.609344 = 40.44 miles

Example 7. (see page 133 in text) Determine the isotropic free space loss at 4 GHz for the shortest downlink path to a geosynchronous satellite (35863 km).

The distance in meters is d = (***35863 km***)(***1000 meters/km***) = 3.5863 × 10^7 meters.
From equation (11), we have a free space loss of

$$L_{dB\ freespace} = 20\log_{10}\left(\frac{4\pi\ f\ d}{c}\right) = 20\log_{10}\left(\frac{4\pi\left(4\times10^9\ Hz\right)\left(3.5863\times10^7\ \text{meters}\right)}{3\times10^8\ m/s}\right)$$
$$= 20\log_{10}\left(\frac{(4\pi)\left(4\times10^9\right)\left(3.5863\times10^7\right)}{3\times10^8}\right) = 20\log_{10}\left(\frac{(4\pi)(4)(3.5863)}{3}\times10^8\right)$$
$$= 20\log_{10}\left(6.0089\times10^9\right) = (20)(9.7788) = 195.6dB$$

Consider satellite (transmitter) and earth station (receiver) antenna gains of 44 and 48 dB, respectively. From equation 15, the total path loss is:

$$L_{dB} = L_{dB\ freespace} - G_{t(dB)} - G_{r(dB)}$$
$$= 195.6\ dB - 44\ dB - 48\ dB = 103.6\ dB$$

Example 8. Given geosynchronous satellite (transmitter) and earth station (receiver) antenna gains of 44 and 48 dB, respectively and a power of 40 watts delivered to the transmitting antenna, compute the received power at the receiving antenna terminals.

From example 7, the path loss is:

$$L_{dB} = 103.6\ dB$$

From (11), we have

$$L_{dB} = 10\log_{10}\left(\frac{P_t}{P_r}\right)$$

Substituting $L_{dB} = 103.6\ dB$ and P_t = 40 watts, we have

$$103.6 = 10\log_{10}\left(\frac{40\ watts}{P_r}\right)$$

Divide by 10 to get:

$$10.36 = \log_{10}\left(\frac{40\ watts}{P_r}\right)$$

Perform antilog to get:

$$10^{10.36} = 2.29\times10^{10} = \frac{40\ watts}{P_r}$$

Solve for received power to get:

$$P_r = \frac{40\ watts}{10^{10.36}} = \frac{40\ watts}{2.29\times10^{10}} = 1.75\times10^{-9}\ watts = 1.75\ nW$$

Example 9. Given geosynchronous satellite (transmitter) and earth station (receiver) antenna gains of 44 and 48 dB, and a power of 40 watts delivered to the transmitting antenna, compute the EIRP of the transmitting antenna.

The power in dBm delivered to the transmitting antenna is

$$P_{t(dBm)} = 10\log_{10}(P_t) = 10\log_{10}(40) = (10)(1.602) = 16.02\ dBm$$

Substituting $P_{t(dBm)} = 16.02\ dBm$ and $G_{t(dB)} = 44\ dB$ into (13) gives us:

$$EIRP_{dBm} = P_{t(dBm)} + G_{t(dB)}$$
$$= 16\ dBm + 44\ dB = 60\ dBm$$

Example 10. Given a receiver with a temperature of 294 °K and a 10 MHz bandwidth, determine the noise power density and the total noise power at he receiver output,

The noise power density is

$$N_0 = kT = \left(1.3803 \times 10^{-23}\right)(294) = 4.058 \times 10^{-21}\ W/Hz$$

and the total noise power at the receiver terminals is:

$$N = BN_0 = \left(4.058\times 10^{-21}\ W/Hz\right)\left(10\times 10^{6}\ Hz\right) = 4.058\times 10^{-14}\ Hz$$

In dBW, this is:

$$N_{dBW} = 10\log_{10}(N) = 10\log_{10}\left(4.058\times 10^{-14}\ W\right) = -133.9\ dBW$$

Example 11. Given a receiver with a temperature of 25 °C receiving a signal at a data rate of 10 Mbps and a signal power of 1 pW, determine the value of $\frac{E_B}{N_0}$ in dB.

The temperature in °K is 25 + 273 = 298°K. The noise power density is

$$N_0 = kT = \left(1.3803 \times 10^{-23}\right)(298) = 4.1133 \times 10^{-21}\ W/Hz$$

The energy per bit is $E_B = \frac{S}{R} = \frac{1\times 10^{-12}}{10\times 10^{6}} = 1\times 10^{-19}$

$$\frac{E_B}{N_0} = \frac{1\times 10^{-19}}{4.1133\times 10^{-21}} = 24.311$$

In dB, $\frac{E_B}{N_0}$ is $\left(\frac{E_B}{N_0}\right)_{dB} = 10\log_{10}\left(\frac{E_B}{N_0}\right) = 10\log_{10}(24.311) = 13.86\ dB$